



**POLITECNICO**  
MILANO 1863

# Dependable Systems

Design for Dependability:  
Fault Detection / Fault Tolerance

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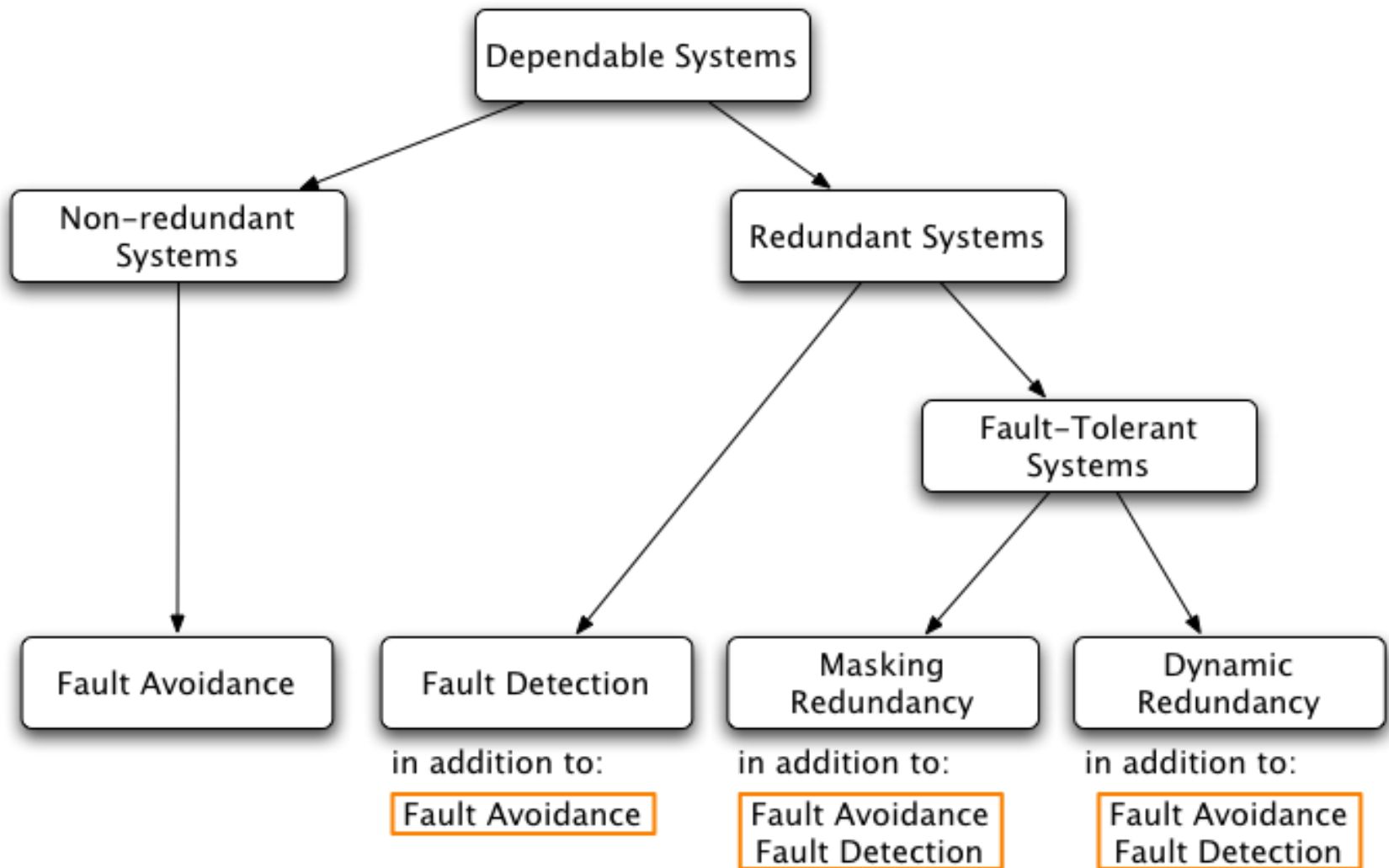
# TOPIC QUESTIONS

How can we detect and handle the occurrence of faults?

What technique is more effective?

What are the costs?

# Fault Management



TECHNIQUES



# Fault management strategies

## Fault avoidance

preventing faults from entering the system during design phase  
(this should be the goal of the entire design process)



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Identification of fault occurrence within the operational system  
during normal functioning



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## Masking Redundancy (Fault tolerance)

The system is equipped with additional resources to tolerate the occurrence of faults; these resources are always on



# Fault management strategies

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## Fault detection

Identification of fault occurrence within the operational system during normal functioning

## Masking Redundancy (Fault tolerance)

The system is equipped with additional resources to tolerate the occurrence of faults; these resources are always on

## Dynamic Redundancy (Fault tolerance)

The system is equipped with additional resources to tolerate the occurrence of faults; these resources can be switched on when needed



# Considered approaches

## Redundancy techniques

- Space/Hardware and time
- Information







# Space/Hardware and Time Redundancy

## TOPIC QUESTIONS

How can I use additional resources/processing to detect/manage faults/errors?

How do I protect processing/data from the outside?

# Space Redundancy

Area (or Space or Hardware) Redundancy

Additional modules, not necessary for performing the “nominal” device functionality, are introduced

From the design point of view, the approach is the one requiring the lowest effort



# Space Redundancy | 2

## Passive redundancy

- fault masking

## Active redundancy techniques

- detection, localization, containment, recovery

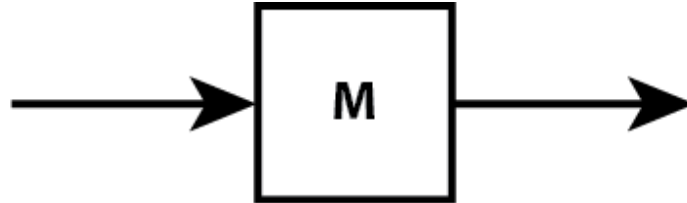
## Hybrid redundancy techniques

- static & dynamic
- fault masking & reconfiguration



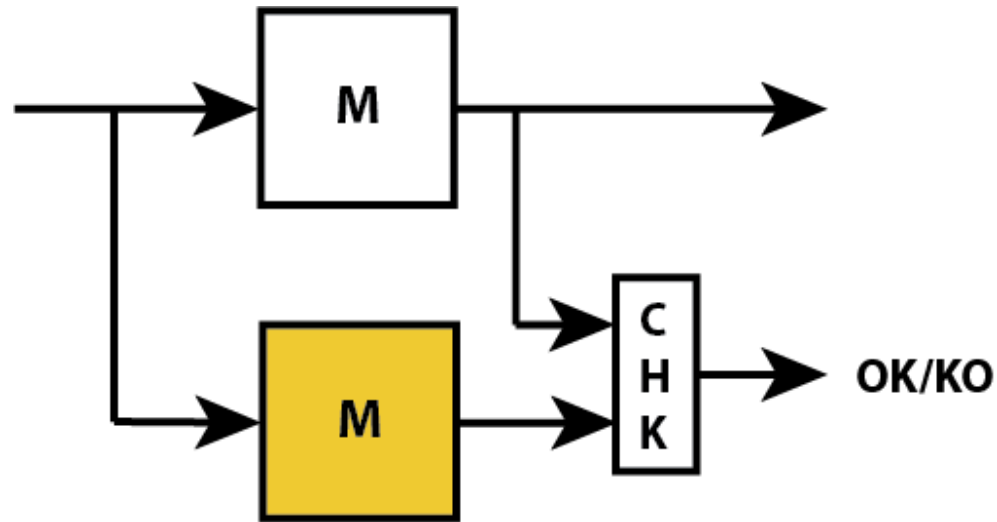
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Two replicas of the nominal module receive the same inputs and their outputs are compared for mismatch



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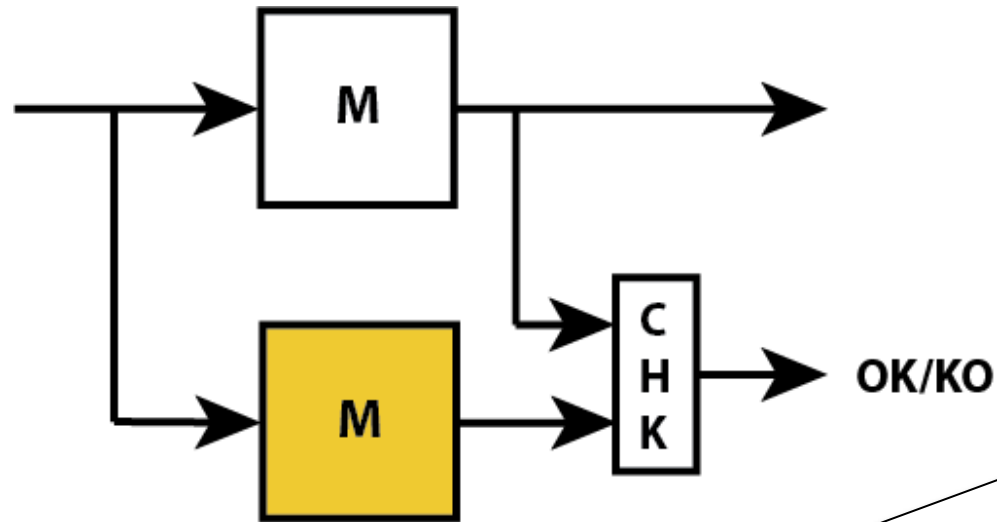


# Space Redundancy | 3

Fault Detection

## Duplication With Comparison (DWC)

Two replicas of the nominal module receive the same inputs and their outputs are compared for mismatch



Under which assumption?

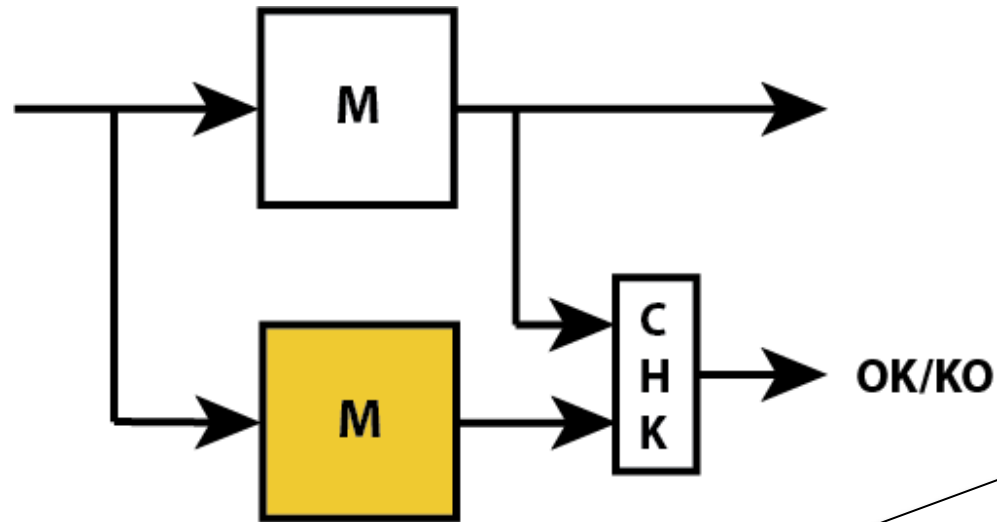


# Space Redundancy | 3

Fault Detection

## Duplication With Comparison (DWC)

Two replicas of the nominal module receive the same inputs and their outputs are compared for mismatch



No faults in the checker!





## Duplication With Comparison (DWC)

Applied at various abstraction levels:

- Module M is a circuit element  
(an adder, a multiplier, ...)
- Module M is a processor  
(Dual processor architectures)



## Duplication With Comparison (DWC)

### Costs/Benefits:

- Area: More than twice the original area
- Checker design: Two-Rail Code Checkers
- Performance: no degradation
- Effort: low



## *N*-Modular redundancy

$N$  replicas of the same module are fed with the same inputs and their outputs are compared and voted to produce the output

Implementation of the  $M$ -out-of- $N$  systems

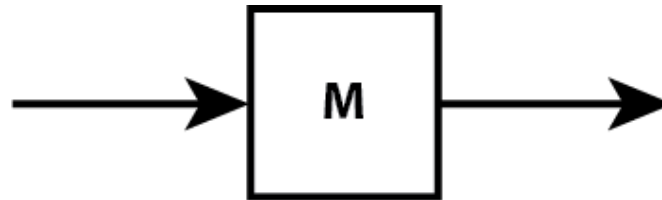
- The system survives if there are  $M$  out of  $N$  working modules



# Space Redundancy | 7

## $N$ -Modular redundancy

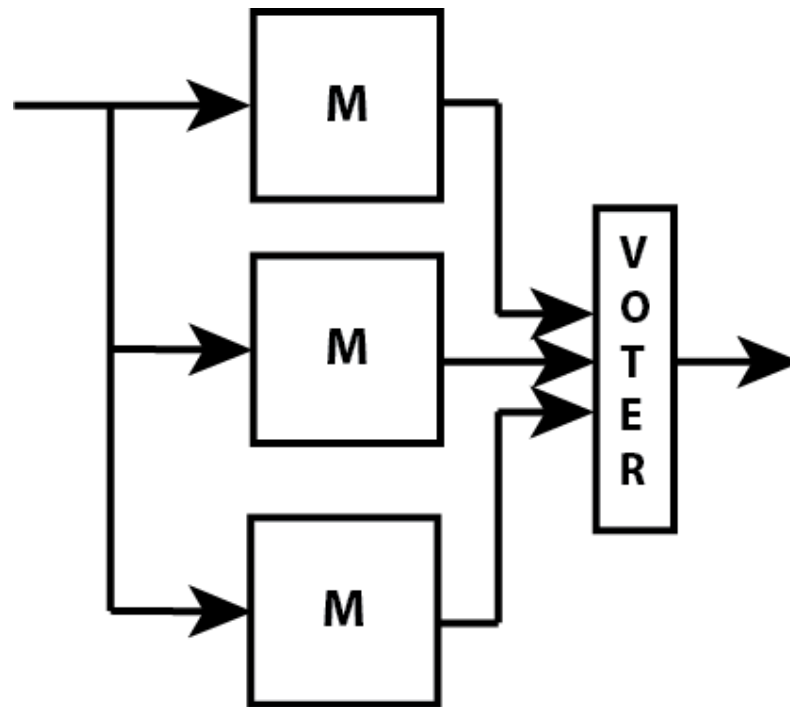
$N = 3$  is the smallest realization, called Triple Modular Redundancy (TMR)



# Space Redundancy | 7

## $N$ -Modular redundancy

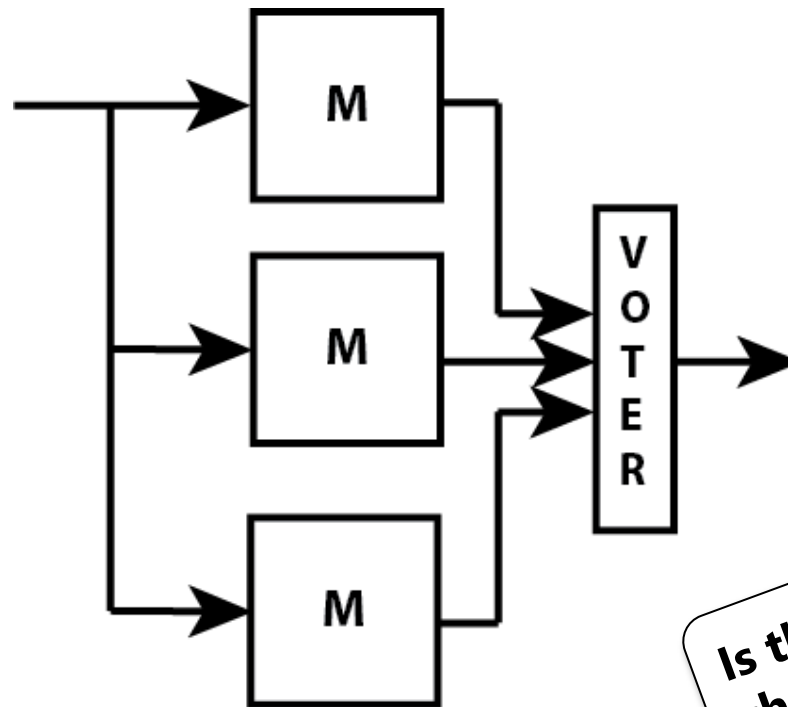
$N = 3$  is the smallest realization, called Triple Modular Redundancy (TMR)



# Space Redundancy | 7

## N-Modular redundancy

$N = 3$  is the smallest realization, called Triple Modular Redundancy (TMR)



*Is the assumption on the checker still required?*



# Space Redundancy | 7

$N = 3$  is the smallest realization, called Triple Modular Redundancy (TMR)

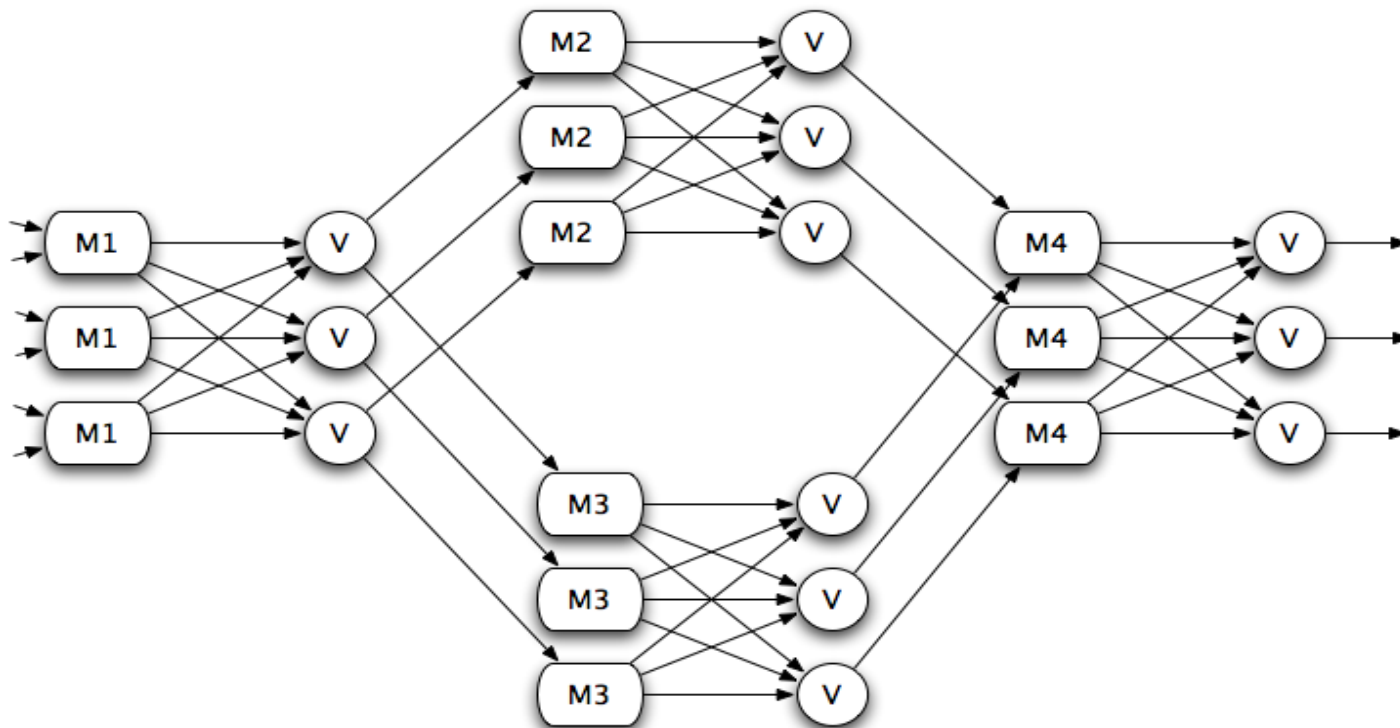
$$R_{TMR}(t) = R_{voter}(t) (3R(t) + R^3(t) - 3R^2(t))$$



# Space Redundancy | 8

## N-Modular redundancy

Application at system level or at module level

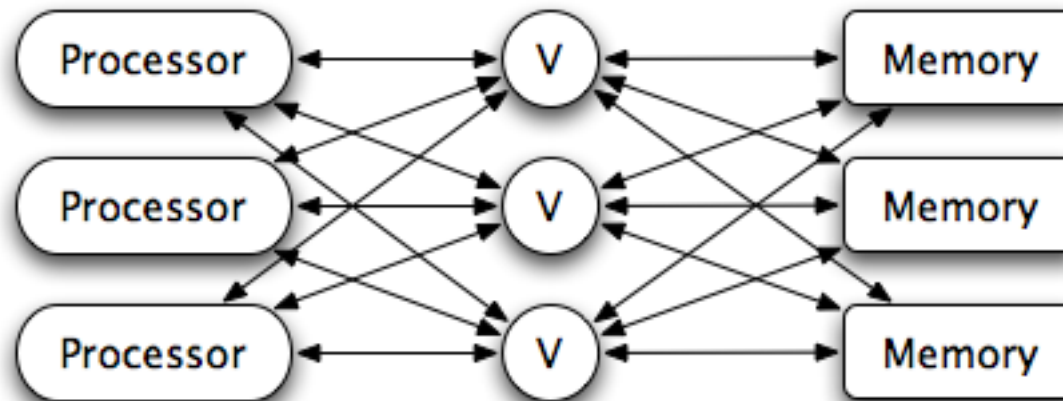




# Space Redundancy | 8

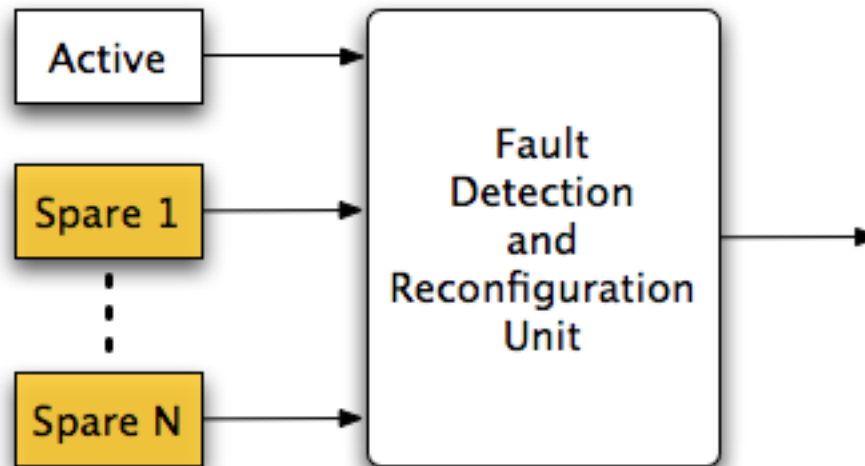
## *N*-Modular redundancy

Application at system level or at module level



## Standby Sparing

When a fault is detected the system reconfigures itself to use one of the redundant modules



# Standby sparing

## Hot standby

- all modules are powered up
- spares can be switched into use immediately after the primary module becomes failed

## Cold standby

- the primary modules are powered up
- the spares are powered down, and then are powered up and switched into use when the primary modules fail

## Warm standby



## Standby sparing | 2

Various fault detection or error detection schemes are used to determine whether a module has become faulty

Fault location is used to determine exactly which module, if any, is faulty



## Standby sparing | 3

The reconfiguration can be viewed as a switch

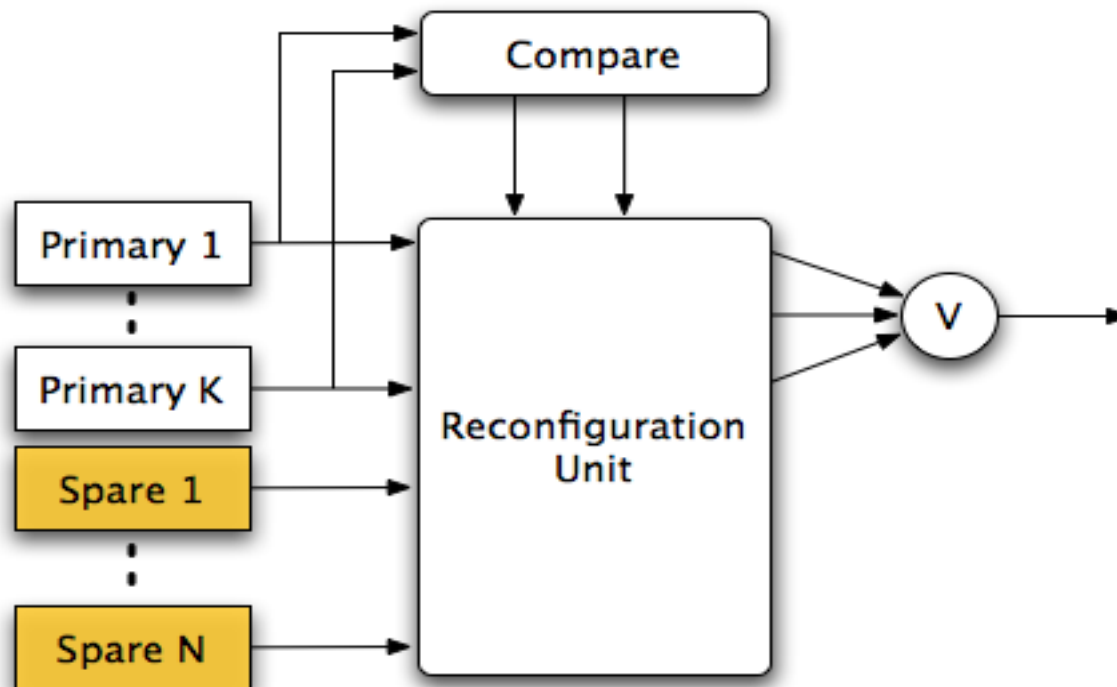
Can bring a system back to full operation after the occurrence of a fault

Require momentary cost in performance when reconfiguration is performed



## Pair-and-a-spare

There is both fault masking and dynamic reconfiguration to cope with faulty modules



## Pair-and-a-spare

Combine the features in **standby sparing and duplication with comparison**

2 modules are operated in parallel at all times and their results are compared to provide the error protection capability

The error signal from the comparison is used to initiate the reconfiguration process (switch) that removes faulty modules and replaces them with spares



# Time redundancy

All the concepts seen so far related to area/hardware redundancy can be applied to *time redundancy*

The application is executed multiple times and the results are checked...

Adopted for temporary, not permanent faults

In not “real-time” systems







# Information Redundancy

# TOPIC QUESTIONS

How can I modify data itself to allow for error detection/correction?

How do I protect (stored) data?

# Information Redundancy

To detect/tolerate errors, additional information is introduced into data, allowing detection/tolerance



# Information Redundancy

To detect/tolerate errors, additional information is introduced into data, allowing detection/tolerance

Information coding

Redundant Array of Independent Disks (RAID) :: for larger data structures

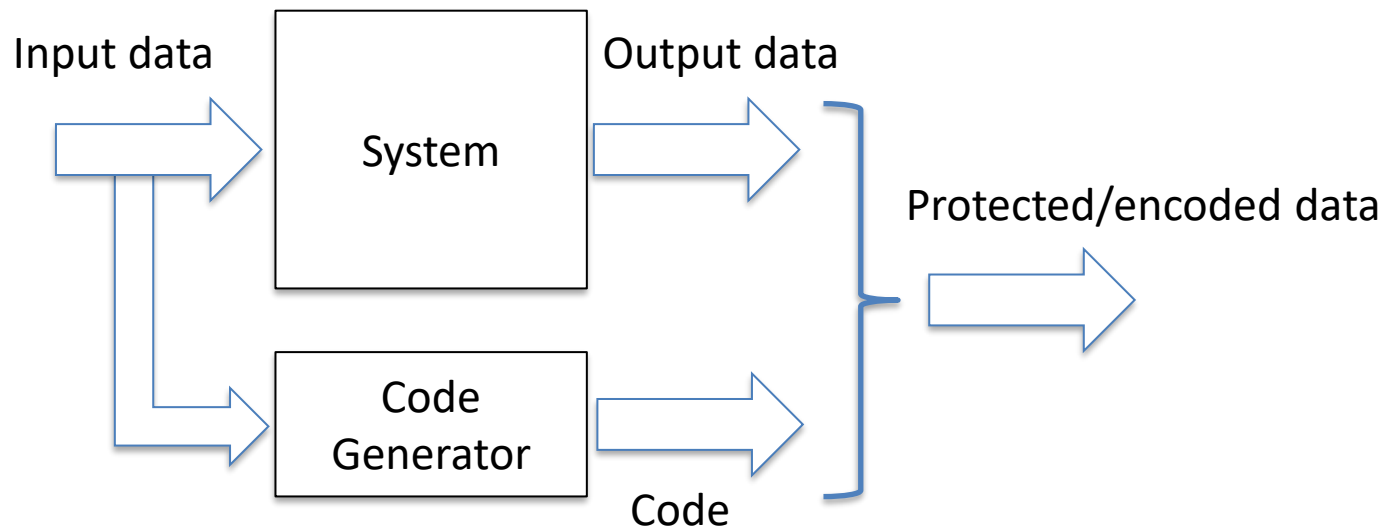




# Information coding

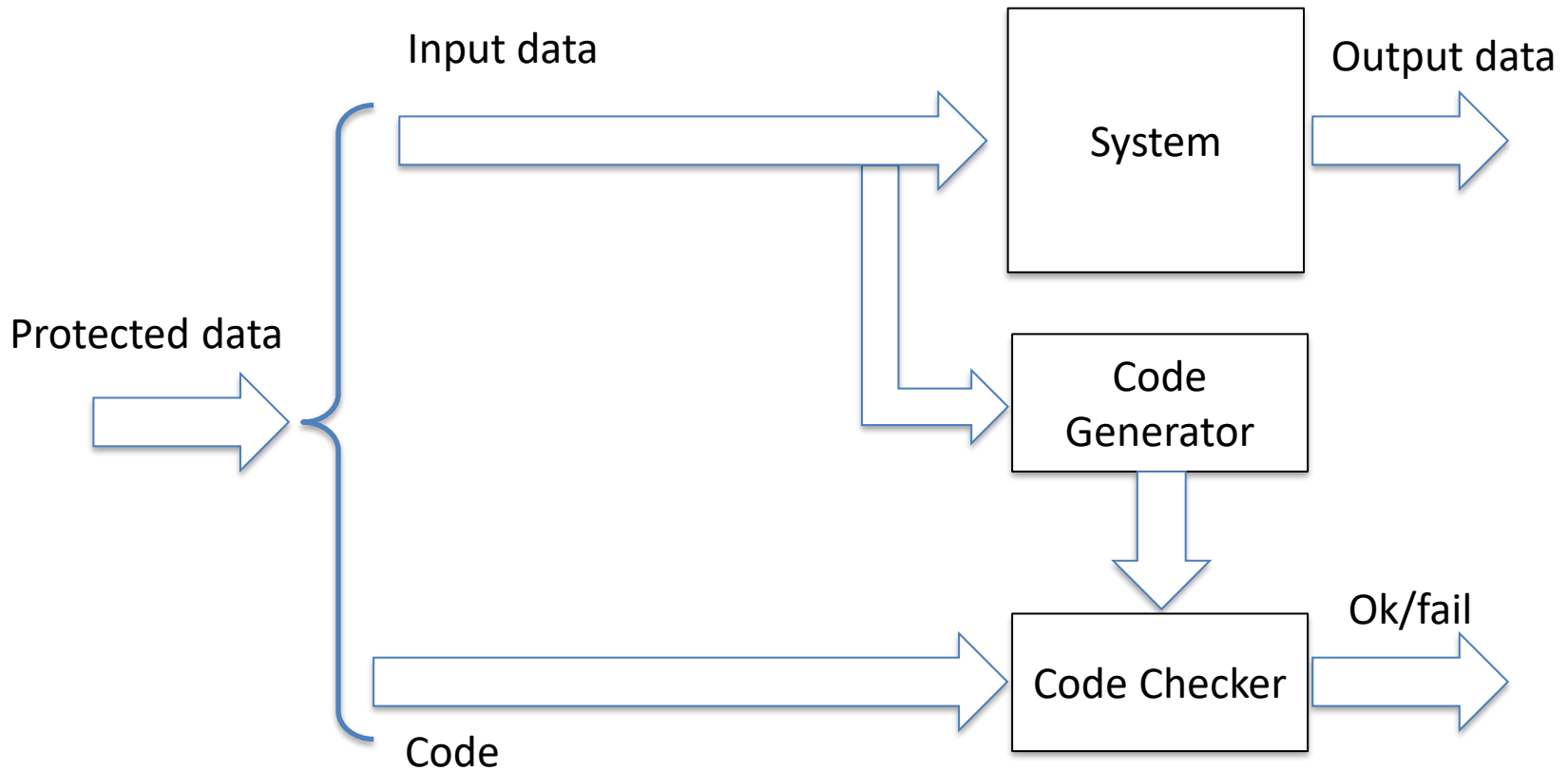
# Information Redundancy

## Data production



# Information Redundancy

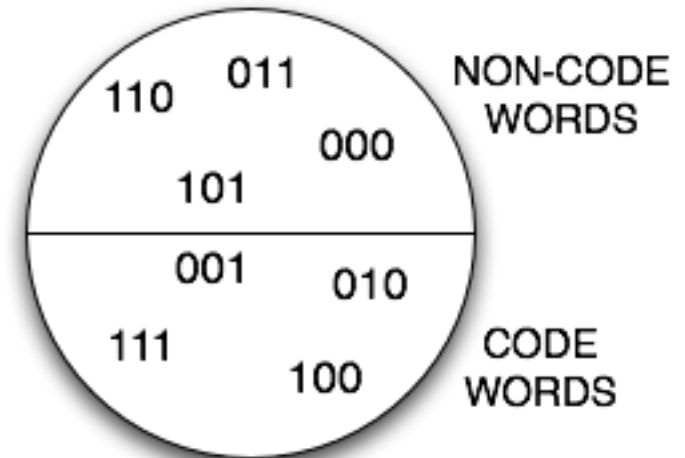
## Data usage



# Information Redundancy | 2

Partition the set of possible configurations in **codewords** and **non-codewords**

- In a fault-free situation data is a codeword
- In a faulty situation data is a non-codeword

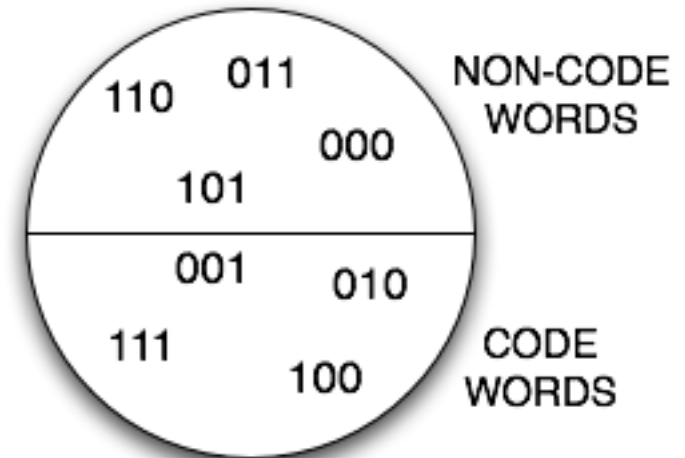




# Information Redundancy | 2

Partition the set of possible configurations in **codewords** and **non-codewords**

- In a fault-free situation data is a codeword
- In a faulty situation data is a non-codeword



**Fault detection**



# Information Redundancy | 3

Associated with the adoption of a code, there must be a synthesis strategy to guarantee that **faults cause only detectable errors**, according to the adopted code

In other words, faults may cause data to be a non-codeword



# Information Redundancy | 4

## Combinational circuits

- Output encoding
- Input encoding  
*allows covering faults on primary inputs*

## Sequential circuits

- Next-state function
- Output function



# Error Detecting Codes (EDC)

Allow the detection of a fault when it produces an error

- Parity Bit
- Berger
- m-out-of-n
- checksum (single/double precision, residue, Honeywell, ...)
- Cyclic codes
- Arithmetic codes



# Parity Bit code

Least redundant bit is used to *code* the remaining bits

**Odd parity bit** is 1 if data contains an even number of ones

**Even parity bit** is 1 if data contains an odd number of ones

Parity on the encoded information

- Odd parity: 100010**1**
- Odd parity: 101010**0**
- Even parity: 100010**0**
- Even parity: 110010**1**



# Parity Bit code

## Examples:

- odd: 1000100**1**
- even: 1000100**0**



# Parity Bit code

## Fault detection:

- odd: 1000100**1**
- even: 1000100**0**
  
- odd: 10**1**0100**1**
- even: 10**1**0100**0**



# Parity Bit code

Fault detection:

- odd: 1000100**1**
- even: 1000100**0**
  
- odd: 10**1**0100**1**
- even: 10**1**0100**0**

Should be 0

**Parity check mismatch!  
Error detected!**

Should be 1





# Parity Bit code

## Fault detection:

- odd: 1000100**1**
- even: 1000100**0**
  
- odd: 10**1**0100**1**
- even: 10**1**0100**0**
  
- odd: 1000100**0**
- even: 1000100**1**

Should be 1

**Parity check mismatch!  
Error detected!**

**Parity check mismatch!  
Error detected!**

Should be 0



# Parity Bit code

Examples:

- odd: 1000100**1**
- even: 1000100**0**

*What about multiple faults?*



# Parity Bit code

## Examples:

- odd: 1000100**1**
- even: 1000100**0**
  
- odd: 100**1**000**1**
- even: 100**1**000**0**

Should be 1

**No parity check mismatch!  
Error undetected!**

Should be 0



# Parity Bit code

## Examples:

- odd: 1000100**1**
- even: 1000100**0**
  
- odd: 100**1**000**1**
- even: 100**1**000**0**
  
- odd: 100**1**100**0**
- even: 100**1**100**1**

Should be 0

**Parity check mismatch!  
Error undetected!**

**No parity check mismatch!  
Error undetected!**

Should be 1



# Parity Bit code

## Examples:

- odd: 1000100**1**
- even: 1000100**0**

- odd: **0**00**1**000**1**
- even: **0**00**1**000**0**

**Parity check mismatch!  
Error detected!**



# Parity Bit code

## Examples:

- odd: 1000100**1**
- even: 1000100**0**

- odd: **0**00**1**000**1**
- even: **0**00**1**000**0**

- odd: 1**1**0**1**100**0**
- even: 1**1**0**1**100**1**

**Parity check mismatch!  
Error detected!**

**Parity check mismatch!  
Error detected!**



# Parity Bit code

Examples:

- odd: 1000100**1**
- even: 1000100**0**

*What about multiple faults?*

**Single parity bit detects an  
odd number of faults**



# Berger code

Code bits encode the number of 0s in the information bits

Costs varies with information size

IN(2)	IN(1)	IN(0)	B1	B0
0	0	0	1	1
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	0	0





# Berger code

Code bits encode the number of 0s in the information bits

Costs varies with information size

IN(3)	IN(2)	IN(1)	IN(0)	B2	B1	B0
0	0	0	0	1	0	0
0	0	0	1	0	1	1
0	0	1	0	0	1	1
0	0	1	1	0	1	0
0	1	0	0	0	1	1
0	1	0	1	0	1	0
0	1	1	0	0	1	0
0	1	1	1	0	0	1
1	0	0	0	0	1	1
1	0	0	1	0	1	0
1	0	1	0	0	1	0
1	0	1	1	0	0	1
1	1	0	0	0	1	0
1	1	0	1	0	0	1
1	1	1	0	0	0	1
1	1	1	1	0	0	0



# Berger code

Code bits encode the number of 0s in the information bits

Costs varies with information size

**Able to detect any number of *unidirectional* faults**

IN(3)	IN(2)	IN(1)	IN(0)	B2	B1	B0
0	0	0	0	1	0	0
0	0	0	1	0	1	1
0	0	1	0	0	1	1
0	0	1	1	0	1	0
0	1	0	0	0	1	1
0	1	0	1	0	1	0
0	1	1	0	0	1	0
0	1	1	1	0	0	1
1	0	0	0	0	1	1
1	0	0	1	0	1	0
1	0	1	0	0	1	0
1	0	1	1	0	0	1
1	1	0	0	0	1	0
1	1	0	1	0	0	1
1	1	1	0	0	0	1
1	1	1	1	0	0	0



# Berger code

Code bits encode the number of 0s in the information bits

Costs varies with information size

***unidirectional faults: only  
0s becoming 1s  
or 1s becoming 0s***

IN(3)	IN(2)	IN(1)	IN(0)	B2	B1	B0
0	0	0	0	1	0	0
0	0	0	1	0	1	1
0	0	1	0	0	1	1
0	0	1	1	0	1	0
0	1	0	0	0	1	1
0	1	0	1	0	1	0
0	1	1	0	0	1	0
0	1	1	1	0	0	1
1	0	0	0	0	1	1
1	0	0	1	0	1	0
1	0	1	0	0	1	0
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1	1	0	0	0	1	0
1	1	0	1	0	0	1
1	1	1	0	0	0	1
1	1	1	1	0	0	0



# Berger code

Examples:

- 0000 **100**

- 000**1** **100**

- **1010** **100**

- 0000 **110**

- 0000 **010**

- 0**100** **101**

- 0**110** **111**

*Number of 0s decreases  
and count keeps unaltered*



# Berger code

Examples:

- 0000 **100**

- 000**1** **100**

- **1010** **100**

*Number of 0s decreases  
and count keeps unaltered*

- 0000 **110**

- 0000 **010**

*Number of 0s keeps  
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- 0000 **010**

*Number of 0s keeps  
unaltered and count increases*

- 0**100** **101**

- 0**110** **111**

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and count increases*



# Berger code

Examples:

- 0000 **100**

- 000**1** **100**

- **1010** **100**

- 0000 **110**

- 0000 **010**

- 0**100** **101**

- 0**110** **111**

*Number of 0s decreases*

*Number of 0s decreases  
and count increases*

***unidirectional faults are  
always detected***



# Berger code

Examples:

- 0010 **011**

- 00**01** **011**

- 0**101** **011**

- 00**01** **001**

- **1**010 **010**

***bidirectional* faults may  
be undetected**





# m-out-of-n code

Given  $m$ -bit data strings,  $n$ -bit keys are added so that the number of 1s (or 0s) in the resulting bit string is constant



# m-out-of-n code

Given m-bit data strings, n-bit keys are added so that the number of 1s (or 0s) in the resulting bit string is constant

d1	d2	d3	k1	k2	k3
0	0	0	1	1	1
0	0	1	0	1	1
0	1	0	0	1	1
0	1	1	0	0	1
1	0	0	0	1	1
1	0	1	0	0	1
1	1	0	0	0	1
1	1	1	0	0	0



# Error Correcting Codes (ECC)

Allow also to correct a non-codeword, identifying the codeword corrupted by the error



# Error Correcting Codes (ECC)

Allow also to correct a non-codeword, identifying the codeword corrupted by the error

Given a non-codeword  $nc$  there is only one codeword  $c$  such that an error transforms  $c$  in  $nc...$

- Hamming
- BCH
- Reed-Solomon & Reed-Muller,
- Binary Golay,
- convolutional & turbo



# Hamming code

Detects double errors (2 erroneous bits), correct single errors

Introduces additional check bits, each one checking a subset of the information bits

Each check bit computes the parity

Several versions exist:

- 1bit data + 2bit check
- 4bit data + 3bit check
- 11bit data + 4bit check
- ...



# Hamming code | 2

Bits of position  $2^k$  are check bits  
(1, 2, 4, 8, 16, 32, 64, ...)

Each parity bit calculates the parity for some of the bits in the code word



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- Parity bit 1 covers all bit positions which have the least significant bit set to 1 => bit 1 (the parity bit itself), 3, 5, 7, 9, etc.



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- Parity bit 2 covers all bit positions which have the second least significant bit set to 1 => bits 2-3, 6-7, 10-11, etc.





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- Parity bit 2 covers all bit positions which have the second least significant bit set to 1 => bits 2-3, 6-7, 10-11, etc.
- Parity bit 4 covers all bit positions which have the third least significant bit set to 1 => bits 4-7, 12-15, 20-23, etc.



# Hamming code | 2

Bits of position  $2^k$  are check bits  
(1, 2, 4, 8, 16, 32, 64, ...)

Each parity bit calculates the parity for some of the bits in the code word

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- Parity bit 4 covers all bit positions which have the third least significant bit set to 1 => bits 4-7, 12-15, 20-23, etc.
- ...
- In general each parity bit covers all bits where the bitwise AND of the parity position and the bit position is non-zero.



# Hamming code | 2

Bits of position  $2^k$  are check bits  
(1, 2, 4, 8, 16, 32, 64, ...)

Bit position	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
Encoded data bits	p1	p2	d1	p4	d2	d3	d4	p8	d5	d6	d7	d8	d9	d10	d11	p16	d12	d13	d14	d15	
Parity bit coverage	p1	x		x		x		x		x		x		x		x		x		x	
	p2		x	x			x	x			x	x			x	x			x	x	
	p4				x	x	x	x					x	x	x	x					x
	p8								x	x	x	x	x	x	x	x					
	p16																x	x	x	x	x

- ...
- In general each parity bit covers all bits where the bitwise AND of the parity position and the bit position is non-zero.



# Hamming code | 3

## Coding (example 4 data bits + 3 check bits)

Data to be coded: 1010

The encoded string will be:

1	2	3	4	5	6	7
$P_1$	$P_2$	1	$P_3$	0	1	0



# Hamming code | 3

## Coding (example 4 data bits + 3 check bits)

Data to be coded: 1010

The encoded string will be:

1	2	3	4	5	6	7
$P_1$	$P_2$	1	$P_3$	0	1	0

$$P_1 = \text{EvenParity}(P_1, 1, 0, 0) \Rightarrow P_1 = 1$$



# Hamming code | 3

## Coding (example 4 data bits + 3 check bits)

Data to be coded: 1010

The encoded string will be:

1	2	3	4	5	6	7
1	$P_2$	1	$P_3$	0	1	0

$$P_2 = \text{EvenParity}(P_2, 1, 1, 0) \Rightarrow P_2 = 0$$



# Hamming code | 3

## Coding (example 4 data bits + 3 check bits)

Data to be coded: 1010

The encoded string will be:

1	2	3	4	5	6	7
1	0	1	$P_3$	0	1	0

$$P_3 = \text{EvenParity}(P_3, 0, 1, 0) \Rightarrow P_3 = 1$$



# Hamming code | 3

## Coding (example 4 data bits + 3 check bits)

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The encoded string is:

1	2	3	4	5	6	7
1	0	1	1	0	1	0





# Hamming code | 3

## Coding (example 4 data bits + 3 check bits)

Data to be coded: 1010

The received string is:

1	2	3	4	5	6	7
1	0	1	1	0	1	0

$$\text{EvenParity}(1,3,5,7)=\text{EvenParity}(1,1,0,0)=0$$

$$\text{EvenParity}(2,3,6,7)=\text{EvenParity}(0,1,1,0)=0$$

$$\text{EvenParity}(4,5,6,7)=\text{EvenParity}(1,0,1,0)=0$$

} No fault occurred



# Hamming code | 3

## Coding (example 4 data bits + 3 check bits)

Data to be coded: 1010

The received string is:

1	2	3	4	5	6	7
1	0	1	1	0	0	0

$$\text{EvenParity}(1,3,5,7)=\text{EvenParity}(1,1,0,0)=0$$

$$\text{EvenParity}(2,3,6,7)=\text{EvenParity}(0,1,0,0)=1$$

$$\text{EvenParity}(4,5,6,7)=\text{EvenParity}(1,0,0,0)=1$$

} Fault occurred in position 6



# Hamming code | 3

## Coding (example 4 data bits + 3 check bits)

Data to be coded: 1010

The received string is:

1	2	3	4	5	6	7
1	1	1	1	0	1	0

$$\text{EvenParity}(1,3,5,7)=\text{EvenParity}(1,1,0,0)=0$$

$$\text{EvenParity}(2,3,6,7)=\text{EvenParity}(1,1,1,0)=1$$

$$\text{EvenParity}(4,5,6,7)=\text{EvenParity}(1,0,1,0)=0$$

} Fault occurred in position 2



# Hamming code | 3

## Coding (example 4 data bits + 3 check bits)

Data to be coded: 1010

The received string is:

1	2	3	4	5	6	7
0	0	1	1	1	1	0

$\text{EvenParity}(1,3,5,7) = \text{EvenParity}(0,1,1,0) = 0$

$\text{EvenParity}(2,3,6,7) = \text{EvenParity}(0,1,1,0) = 0$

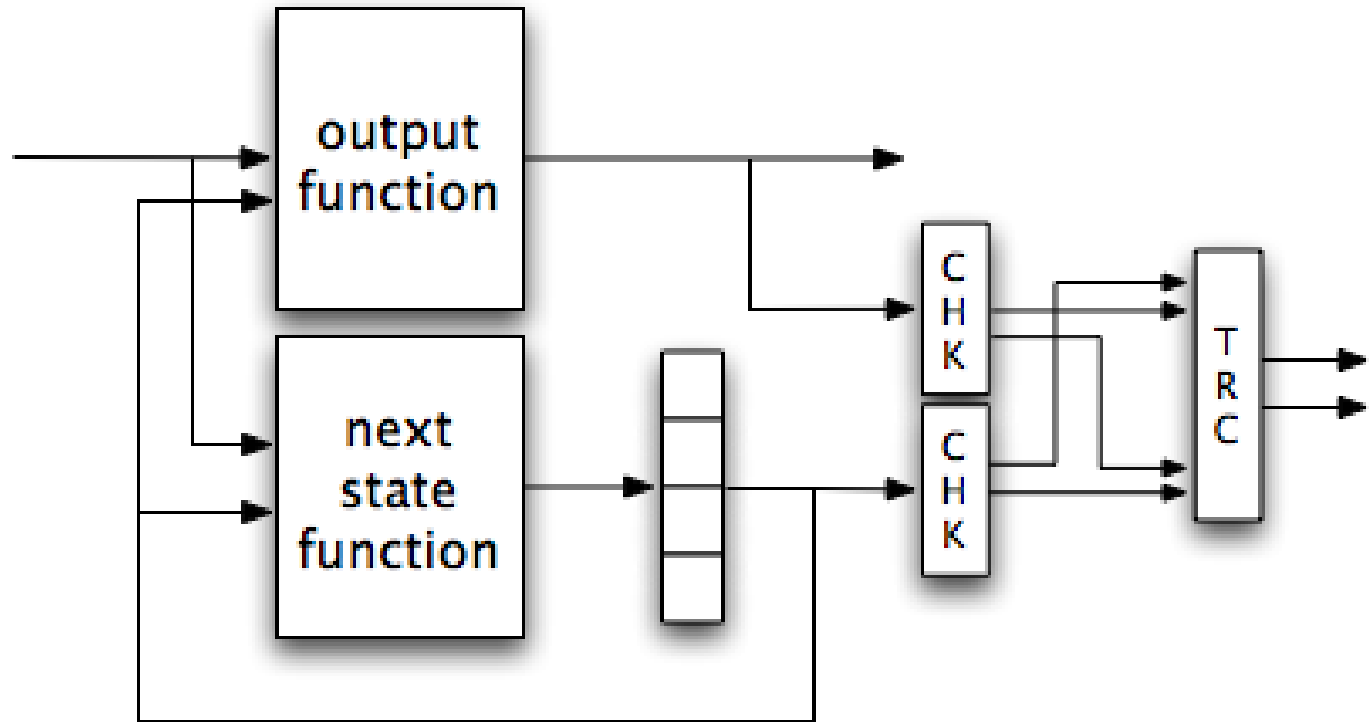
$\text{EvenParity}(4,5,6,7) = \text{EvenParity}(1,1,1,0) = 1$

} Double faults detected  
but not corrected



# Sequential circuits

Encoding of the next-state and of the output function of sequential circuits





# Redundant Array of Independent Disks (RAID)

# RAID

Multiple Hard Disks are deployed to introduce data redundancy

Several versions of RAID have been proposed



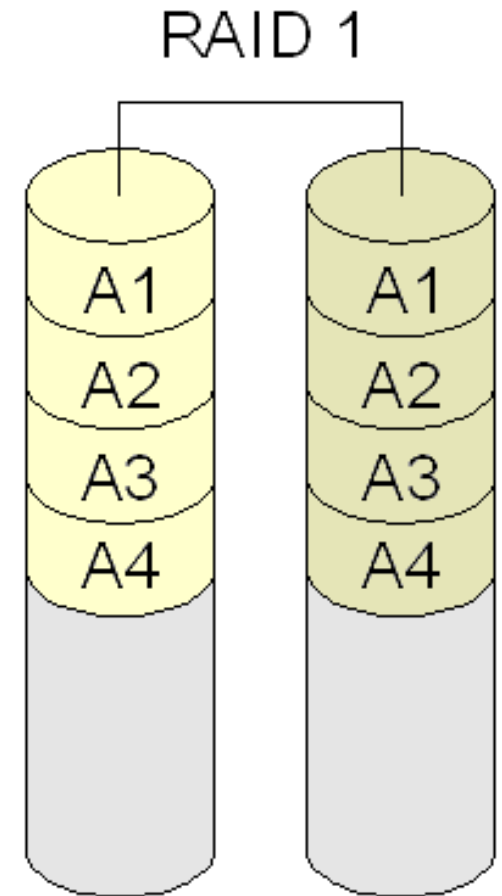
# RAID 1

Simply two (or more) mirrored hard disks

The more mirrors, the more reliability

Low scalability

The slowest disk affects the speed of the entire system

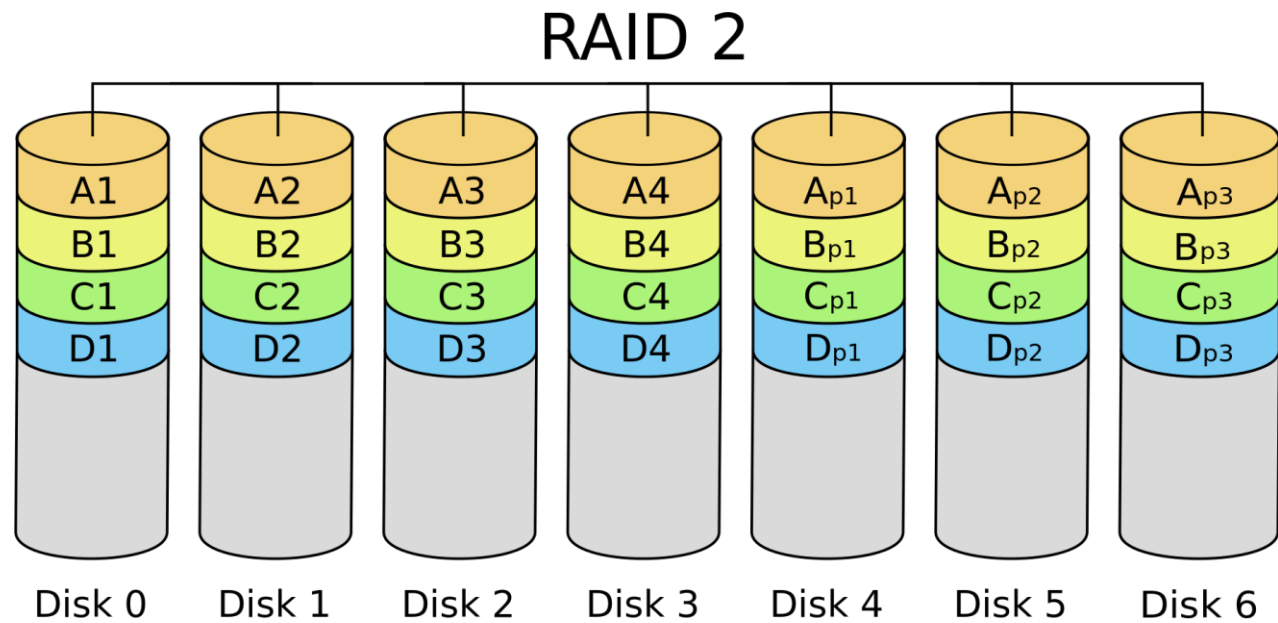




# RAID 2

Data is bit-wise partitioned among disks

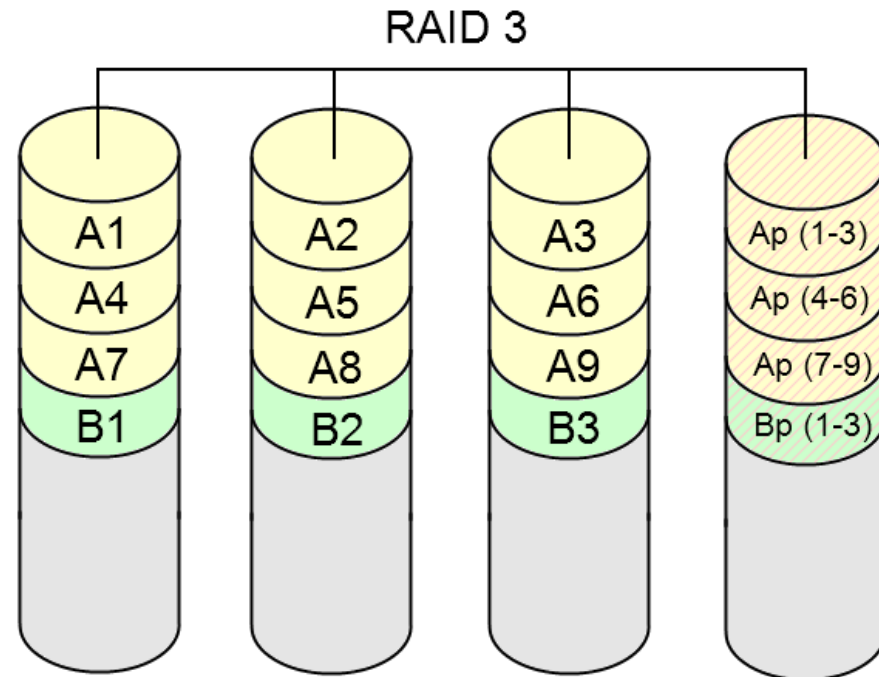
Additional disks for hamming code bits are added



# RAID 3

Data is Byte-wise partitioned among disks

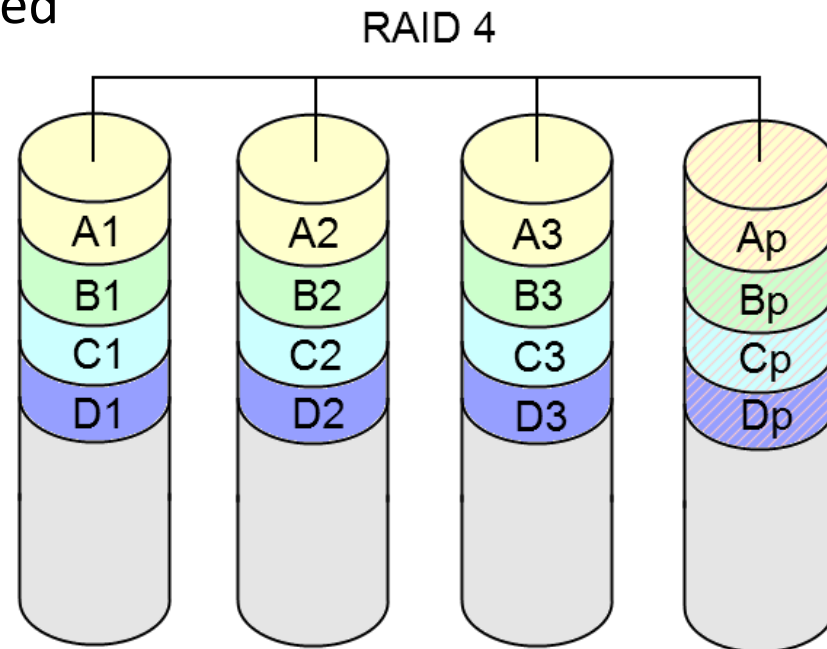
An additional disk for parity is added



# RAID 4

Like RAID 3 but data is block-wise partitioned among disks

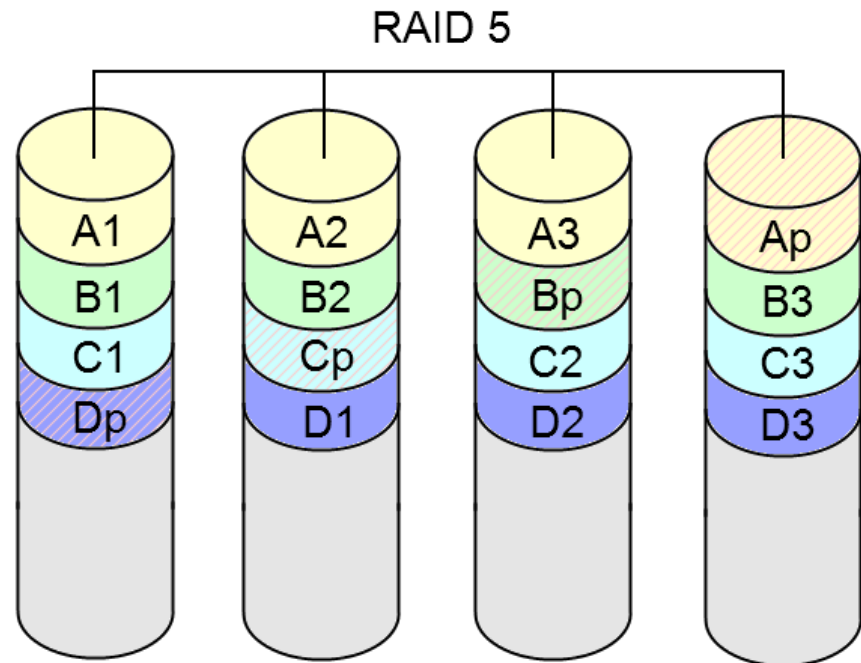
An additional disk for parity is added



# RAID 5

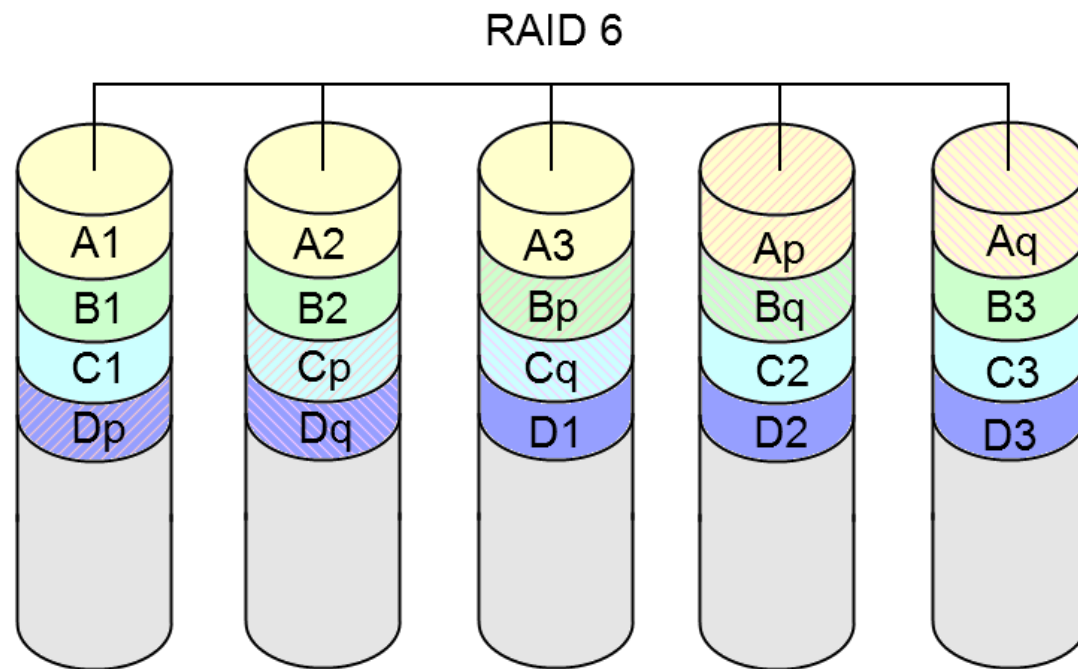
Like RAID 4, data is block-wise partitioned among disks

Parity is distributed among disks



# RAID 6

Like RAID 5 but with double parity distributed among disks



## TOPIC QUESTIONS

How can we handle the occurrence of faults?

Is there a more appropriate kind of redundancy than others?

What technique is more effective?

What are the costs?

## TOPICS

Key: Redundancy

Techniques exploiting different kinds of redundancy

Relevant aspects: costs, performance, power, fault coverage